Complex Number
জটিল রাশি

Definition of Complex Number

If x *and* y *are any two real numbers* (বাস্তব সংখ্যা) then

$z = x + iy$ is called a **complex number** (জটিল রাশি) where $i = \sqrt{-1}$

We can also express the complex number, z as

$z = x + iy = (x, y)$

i is called Fundamental Imaginary Unit (কাল্পনিক সংখ্যার মূল একক)।

$i^2 = -1$

$z = x + iy$ where x is called Real Part (বাস্তব অংশ) of z and y is called Imaginary Part (কাল্পনিক অংশ) of z. We write x and y as following:

$x = Re(z)$ and $y = Im(z)$

When $x \neq 0$ and $y = 0$, then z is called "*Purely Real*" (বিশুদ্ধ বাস্তব) as in this case, $z = x$

When $x = 0$ and $y \neq 0$, then z is called "*Purely Imaginary*" (বিশুদ্ধ কাল্পনিক) as in this case, $z = iy$

Equality of Two Complex Numbers (দুটি জটিল রাশির সমতা)

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are any two complex numbers, where x_1, y_1, x_2, y_2 are real numbers.

$z_1 = z_2$ if and only if (যদি এবং কেবলমাত্র যদি) $x_1 = x_2$ and $y_1 = y_2$

Conjugate Complex Number
(অনুবন্ধী জটিল রাশি)

Let $z = x + iy$ is a complex number. Then $x - iy$ is called conjugate

complex number of z and it is written as $\bar{z}$ i.e. $\bar{z} = x - iy$

Let z_1 and z_2 are two complex numbers. Then

$$\bar{\bar{z_1}} = z_1, (\overline{z_1 + z_2}) = \overline{z_1} + \overline{z_2}\ , (\overline{z_1 \cdot z_2}) = \overline{z_1} \cdot \overline{z_2}, \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

Addition, Subtraction, Multiplication & Division of two Complex Numbers

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are any two complex numbers,

where x_1, y_1, x_2, y_2 are real numbers.

- $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2) = (x_1 + x_2, y_1 + y_2)$
- $z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2) =$
  $(x_1 - x_2, y_1 - y_2)$
- $z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$
  as $i^2 = -1$

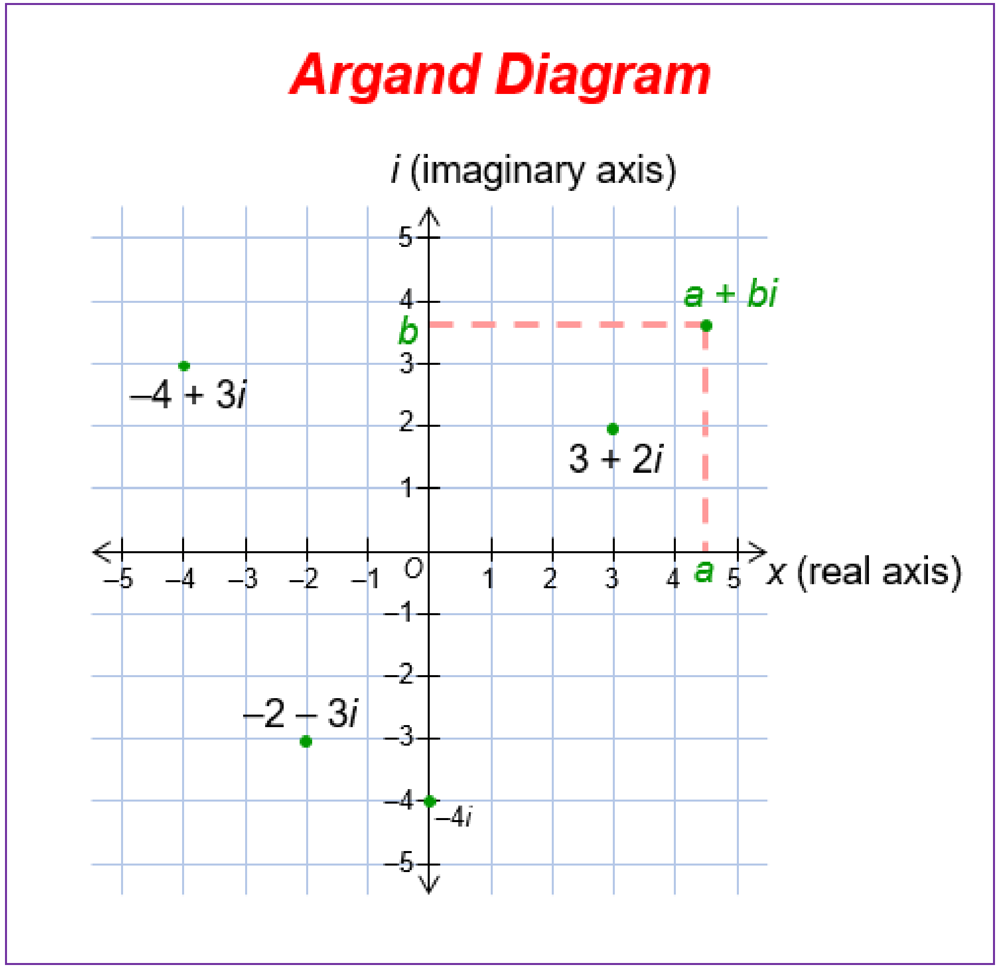
$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$= \frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}$$

$$= \frac{x_1x_2 - i^2y_1y_2 + i(x_2y_1 - x_1y_2)}{x_2^2 - i^2y_2^2}$$

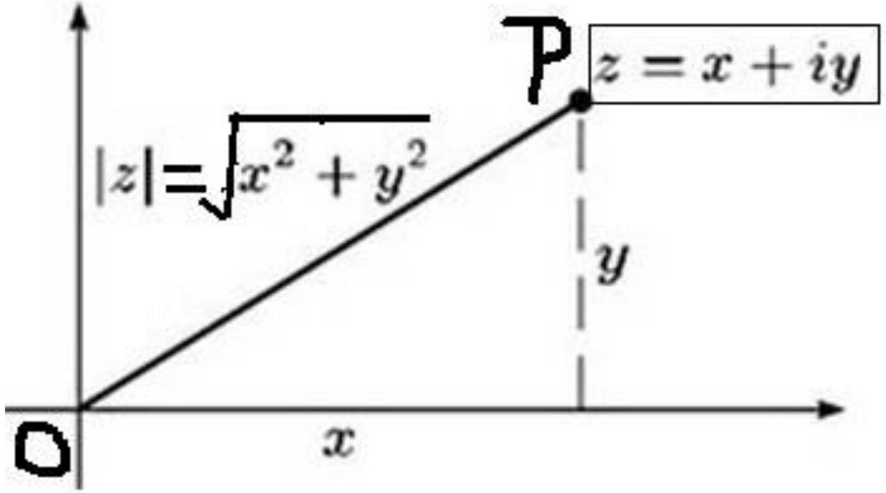
$$= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

Argand Diagram



A Complex Number doesn't appear anywhere on a real number line and so wouldn't appear in coordinates in xy-plane. This problem was solved by Jean-Robert Argand. An Argand diagram uses the real and imaginary parts of a complex number as analogues of x and y in the Cartesian plane. The area of an Argand diagram is called the complex plane by mathematicians.

MODULUS OF A COMPLEX NUMBER



Let O be the origin of argand diagram & P be a point on complex plane, which represents the complex number $z = x + iy = (x, y)$

Then the distance between O & P = $|\overline{OP}| = \sqrt{x^2 + y^2}$ is called the modulus of complex number of z and it is denoted by $|z|$ or *mod(z)*

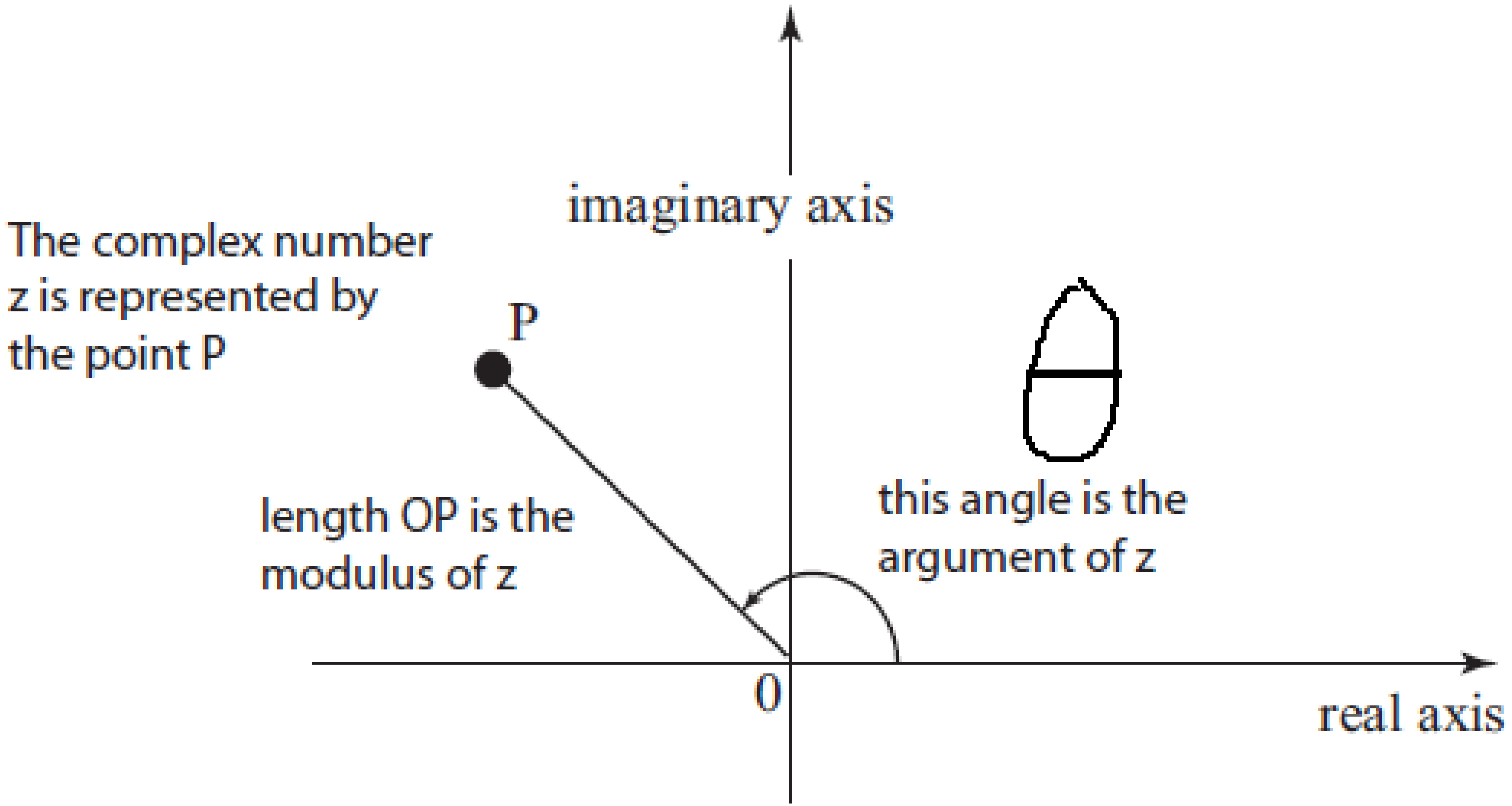
$$\therefore |z| = |\overline{OP}| = \sqrt{x^2 + y^2}$$

Remember that $|z|$ is always positive real number.

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad z\bar{z} = |z|^2$$

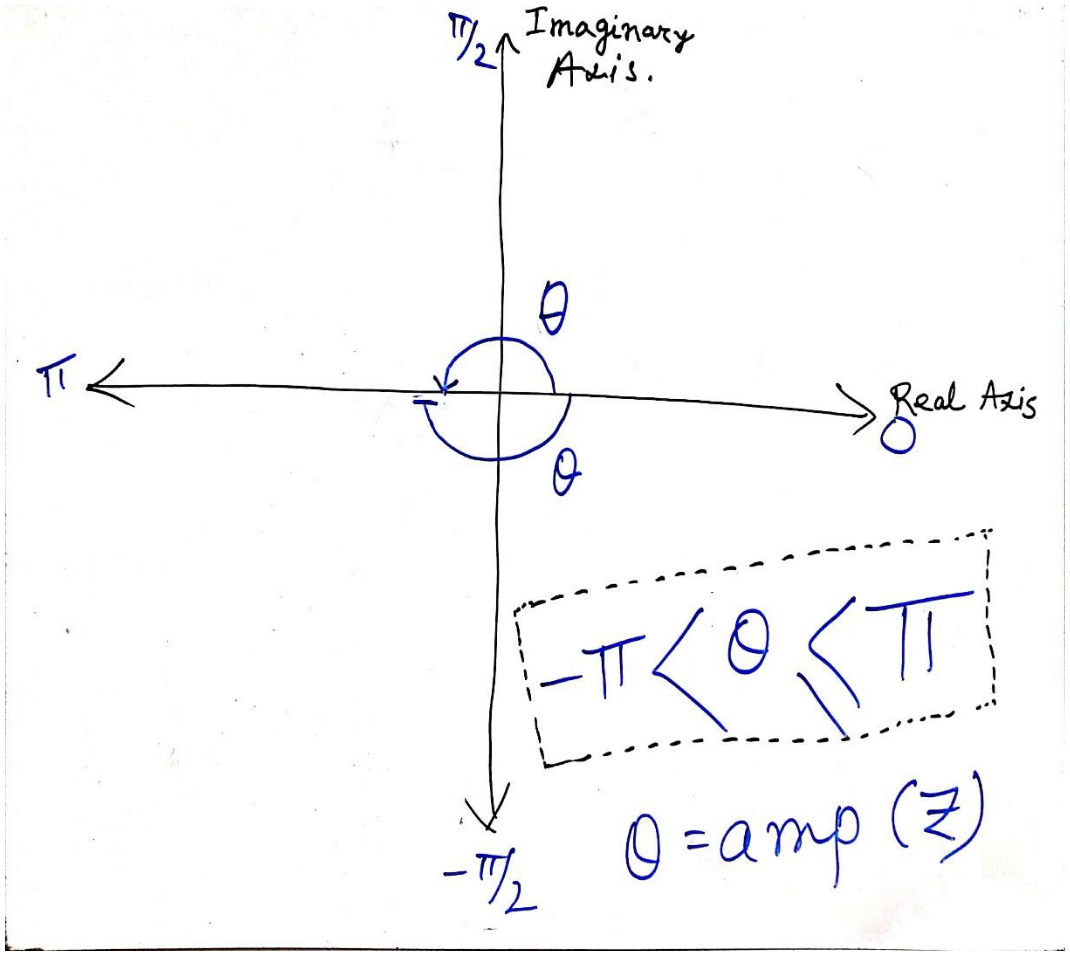
Amplitude / **Argument** of a Complex Number



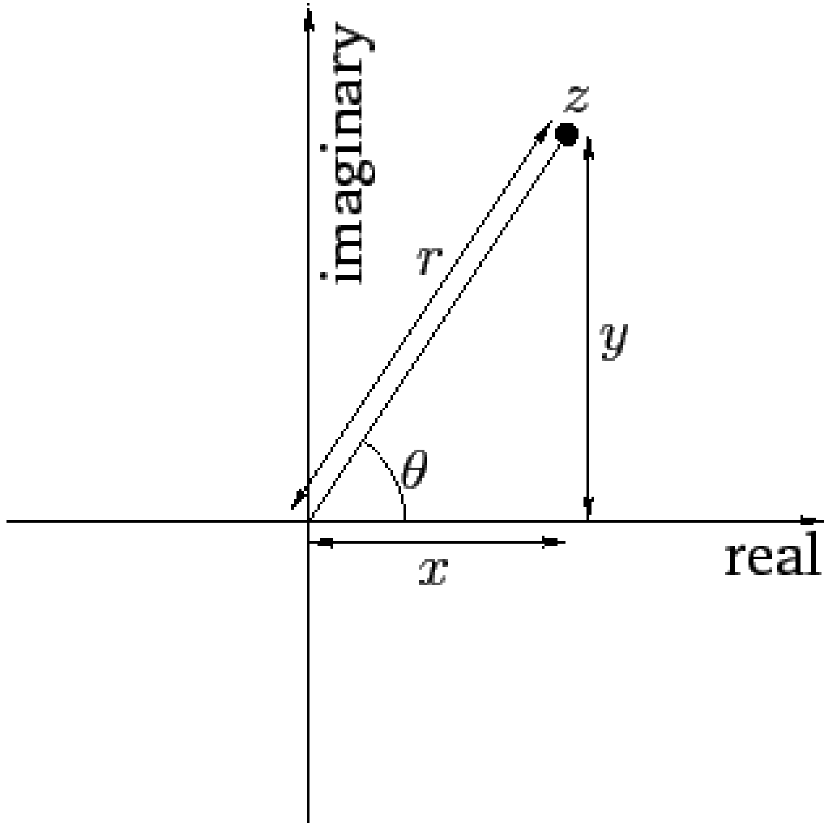
Amplitude / Argument of a Complex Number

The angle is called the Amplitude or Argument of z and is denoted by **Arg (z)** or **Amp (z)**, and has *infinite values*. However, the unique value of θ lying between $-\pi < \theta \leq \pi$, is called principal value of Arg (z) / Amp (z) , and is denoted by **arg (z)** or **amp (z)**

Amp (z) = amp (z) $+2n\pi$ where n is integer.



Modulus-Amplitude Form of a **Complex Number**



Modulus of z = r . From figure,

$\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$

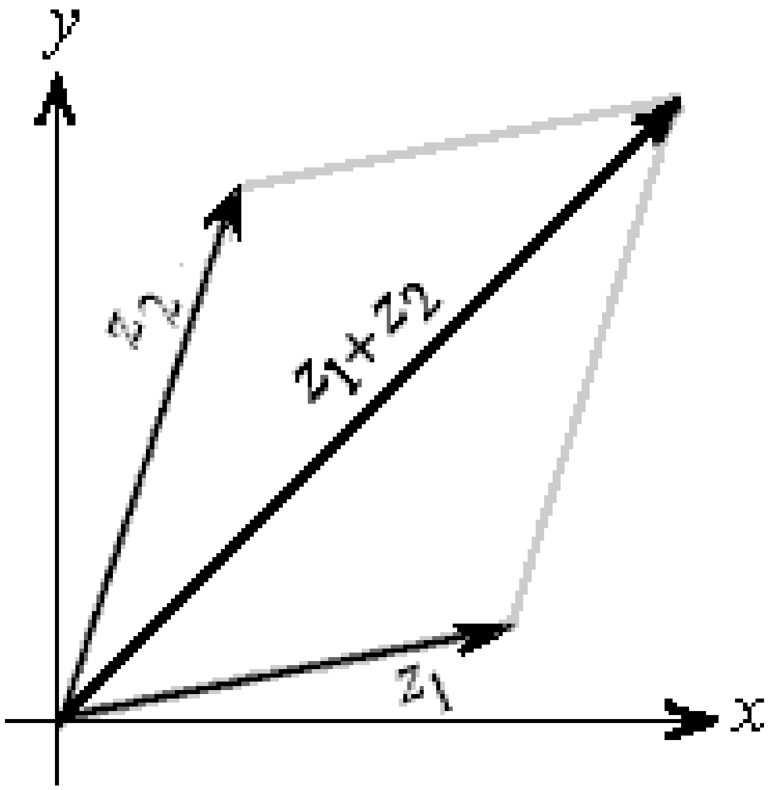
i.e. $x = r\cos\theta$ and $y = r\sin\theta$ where θ is amplitude of z. We know that

$z = x + iy = r\cos\theta + i \cdot r\sin\theta$

$\boxed{z = r(\cos\theta + i\sin\theta)}$. This form of z is known as modulus-amplitude form as it contains both the modulus (r) and amplitude (θ).

Geometrical Representation of **Complex Numbers**

Geometrically, addition of two complex numbers z_1 and z_2 can be visualized as addition of the vectors by using the "*parallelogram law*".

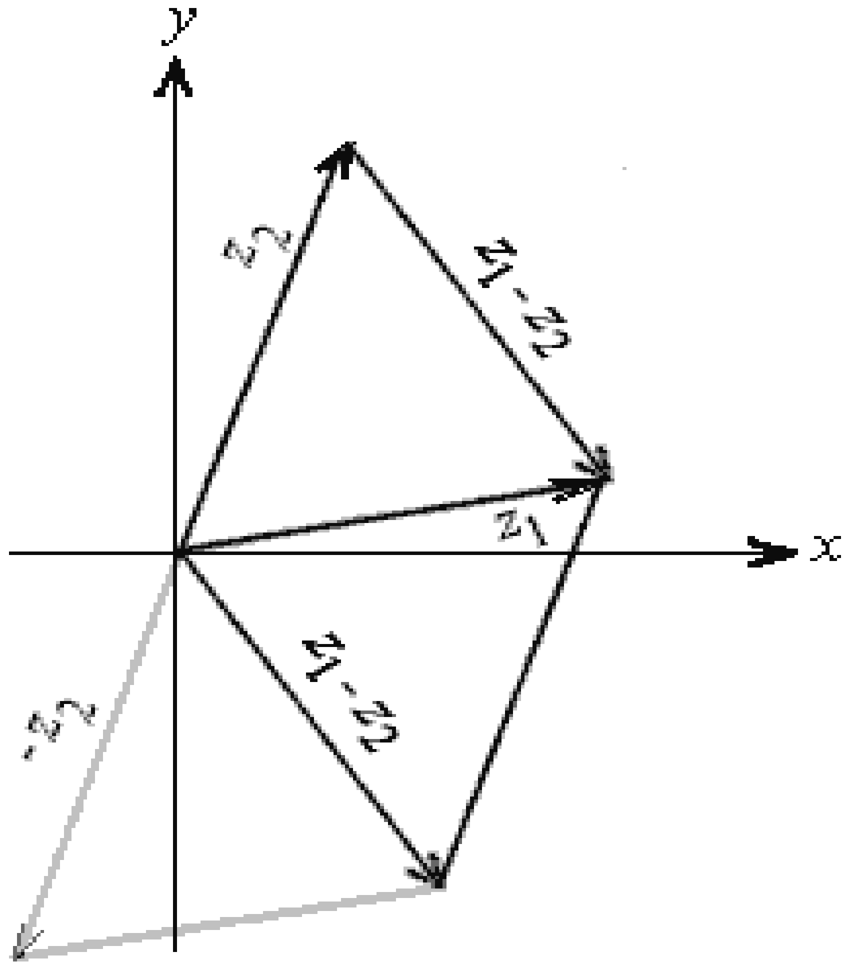


The vector sum z_1 and z_2 is represented by the diagonal of the parallelogram formed by the two original vectors.

From figure, $|z_1 + z_2| \leq |z_1| + |z_2|$

Geometrical Representation of **Complex Numbers**

Geometrically, subtraction of two complex numbers z_1 and z_2 can be visualized as addition of the vectors z_1 and $-z_2$ by using the "*parallelogram law*".



From figure, $|z_1 - z_2|$ represents the distance between two points, which are represented by complex numbers z_1 and z_2.

From figure, $|z_1 - z_2| \leq |z_1| + |z_2|$

Difference between **REAL NUMBER & COMPLEX NUMBER**

In mathematics, the law of **trichotomy** states that every real number is either positive, negative, or zero.

So, if a & b be any two real numbers, then from the above law, we can say that either $a > b$ or $a < b$ or $a = b$

But in Complex number, this law is not holds good. E.g. if z_1 and z_2 are two complex numbers, we can't say that $z_1 > z_2$ or $z_1 < z_2$

Because, in complex number, $i = \sqrt{-1}$ is neither positive nor negative nor zero. Why?

Let, $i > 0$

Multiplying both sides by i, we get

$i \cdot i > 0 \cdot i \Longrightarrow -1 > 0$ which is not possible. So $i \ngtr 0$

Let, $i < 0$

Multiplying both sides by i, we get

$i \cdot i > 0 \cdot i \Longrightarrow -1 > 0$ which is not possible. So $i \nless 0$

So, *law of trichotomy* not holds for Complex Numbers.

De Moivre's Theorem

From De Moivre’s theorem, we can say that if ‘n’ be any rational number, positive or negative, then

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$