# Matrix

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1) If $A = [a_{ij}]$ is a matrix of order $2 \times 2$ where $a_{ij} = i + 2j$ then find the matrix $A$.

2) If $A = [a_{ij}]$ is a matrix of order $2 \times 2$ where $a_{ij} = \frac{(i+j)^2}{2}$, then find the matrix $A$.

3) If $A = (a_{ij})_{n \times n}$ is a square matrix where $a_{ij} = i^2 - j^2$, then matrix $A$ is -- (a) zero matrix, (b) unit matrix, (c) symmetric matrix, (d) skew-symmetric matrix

4) If $A = \begin{pmatrix} 3x & x-1 \\ 2x+3 & x+2 \end{pmatrix}$ is a symmetric matrix, then find the value of $x$.

5) If $A = \begin{bmatrix} 0 & x & 7 \\ -2 & z & 3 \\ y & w & 0 \end{bmatrix}$ is a skew-symmetric matrix,then find the value of $x$, $y$, $z$

6) Show that $A - A^T$ is a skew-symmetric matrix where $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

7) If $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ then show that $A + A^T$ is a symmetric matrix.

8) If $A = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$ then $A^T A$ is equal to - (a) $A$ (b) $-A$ (c) $I$ (d) $-I$

9) If $A$ is a square matrix, then $AA^T + A^T A$ is -- (a) unit matrix (b) zero matrix (c) symmetric matrix (d) skew-symmetric matrix

10) If $A$ be a square matrix, then which of the following is false: (a) $(A^T)^T = A$, (b) $A + A^T$ is symmetric, (c) $A - A^T$ is skew-symmetric, (d) $(AB)^T = B^T \cdot A^T$

11) If $A$ & $B$ be two symmetric matrices, then the matrix $AB$ will be symmetric if - (a) $AB = O$ (b) $AB = BA$ (c) $|AB| = 0$ (d) None of these.

12) If $A$ & $B$ both are symmetric matrices, then the matrix $ABA$ is -- (a) symmetric (b) skew-symmetric (c) diagonal (d) none of these

13) If a matrix $A$ is both symmetric & skew-symmetric, then $A$ should be -- (a) diagonal matrix, (b) zero matrix, (c) square matrix, (d) none of these.

14) If the matrices $A$, $B$ are such that $AB = A$ and $BA = B$. Then $B^2$ is -- (a) $B$, (b) $A$, (c) $I$, (d) $O$

15) If the matrix $A$ is proper orthogonal, then value of $|A|$ is -- (a) 0, (b) 1, (c) 2, (d) 3

16) If $\omega$ is the imaginary cube root of 1 and $A = \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}$, $B = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$, then $(A+B)^{47}$ is -- (a) $-I_2$, (b) $\omega I_2$, (c) $-\omega^2 I_2$, (d) $-\omega I_2$

17) If $A = \begin{pmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{pmatrix}$ is a singular matrix, then value of $x$ is -- (a) $0$, (b) $1$, (c) $3$, (d) $-3$

18) $A$ and $B$ are two matrices such that $A \cdot B = O$ (where $O$ is zero matrix). Can we deduce that either $A$ or $B$ is a zero matrix? Illustrate by an example.

19) $A$ is a matrix of order $2 \times m$ and $B$ is a matrix of order $3 \times n$ . If $A \cdot B$ is defined and its order is $p \times 4$, then find the value of $m, n, p$.

20) For any two square matrices $A$ & $B$, when the matrix equation $A^2 - B^2 = (A+B)(A-B)$ holds true?

21) If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, then show that $(A+B)^2 \neq A^2 + 2AB + B^2$

22) If $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$ then show that $AB \neq BA$

23) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, then show that $f(A) = 0$

24) $A$ is a square matrix such that $A^2 = A$. Then find the value of $(I+A)^3 - 7A$

25) If $A$ is a symmetric matrix, then $A^n$ (where $n$ is positive integer) is ______ matrix.

26) The matrix $\begin{bmatrix} cos\ x & sin\ x \\ -sin\ x & cos\ x \end{bmatrix}$ is - (a) symmetric & singular; (b) unit & skew-symmetric; (c) Unit & orthogonal; (d) Non-singular & orthogonal matrix.

27) If $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, then $A^5$ is -- (a) $5A$, (b) $10A$, (c) $16A$, (d) $32A$

28) If the matrix $\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & x \end{bmatrix}$ is non-singular, then value of $x$ is -- (a) $-2 \leq x \leq 2$, (b) Any real number except $\pm 2$, (c) $x \geq 2$, (d) $x \leq -2$

29) If $A = \begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + \cdots + x^{20}$, then show the value of $f(A)$ is -- (a) $\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$

30) Find a matrix $X$ such that $2A + B + X = 0$ where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

31) If $A = \begin{pmatrix} -1 & 5 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix}$ then show that $(A+B)^T = A^T + B^T$ and $(AB)^T = B^T \cdot A^T$

32) If $2A - 3B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$ and $3A + 2B = \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$ , then find the matrix $A$ and $B$.

33) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then prove that $A \cdot A^T = I$. Hence find $A^{-1}$

34) Show that $A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ is a proper orthogonal matrix, and hence find $A^{-1}$.

35) If $\begin{pmatrix} x+y & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & x-z \\ 2x-y & 0 \end{pmatrix}$ then find the value of $z$.

36) If $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ then find the value of $x, y$

37) If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ then find the values of $k, a, b$.

38) If the matrices $A$ & $B$, given by $A = \begin{bmatrix} x+y & y-z \\ 5-t & z+x \end{bmatrix}$, $B = \begin{bmatrix} t-x & z-t \\ z-y & x+z+t \end{bmatrix}$, are equal, then find the values of $x, y, z, t$

39) If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and $A^2 = I$ then find the value of $x$.

40) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$, then find the value of $k$.

41) If $A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$ and $A^2 = -xI + yA$ (where $I$ is the identity matrix). Then find the value of $x$ and $y$.

42) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the value of $a$ and $b$.

43) If $A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 4 \\ -1 & 7 \end{pmatrix}$ then find the matrix $3A^2 - 2B + I$

44) If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then prove that $(A-2I)(A-3I) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

45) If $A = \begin{bmatrix} 4 & 2i \\ i & 1 \end{bmatrix}$ then show that $(A-2I)(A-3I) = O$ where $O$ & $I$ are zero & unit matrices. $i = \sqrt{-1}$

46) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then prove that $A^2 - (a+d)A = (bc - ad)I$

47) If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and $A^2 = B$, then find the value of $\alpha$ (if possible). Give reason for your answer.

48) If $\alpha$ and $\beta$ are two roots of the equation $x^2 + x + 1 = 0$ then find the matrix $A$ such that $A = \begin{bmatrix} 1 & \beta \\ \alpha & \alpha \end{bmatrix} \times \begin{bmatrix} \alpha & \beta \\ 1 & \beta \end{bmatrix}$

49) If $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ then find the value of $x$ and $y$.

50) If $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $AX = 3B + 2C$, then find the matrix $X$

51) If $A$, $B$, $C$ are 3 given matrices such that $A = \begin{pmatrix} 3 & 5 \\ 2 & a \end{pmatrix}$, $B = \begin{pmatrix} 4 & b \\ 2 & 9 \end{pmatrix}$ and $C = \begin{pmatrix} 22 & 14 \\ a & -1 \end{pmatrix}$, find $a$ and $b$ such that $A \cdot B = C^T$.

52) Find $A^{100}$ if $A = \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}$, where $\omega$ is the imaginary cube root of 1.

53) If $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ then show that $A^8 = 128\ B$

54) If $A=\begin{bmatrix}1&1\\1&1\end{bmatrix}$ and $n\in\mathbb{N}$, then show that $A^n=2^{n-1}A$

55) If $A=\begin{pmatrix}3&-4\\1&-1\end{pmatrix}$, then show that $A^n=\begin{pmatrix}1+2n&-4n\\n&1-2n\end{pmatrix}$ where $n\in\mathbb{N}$

56) If $A=\begin{bmatrix}1&0\\\frac{1}{2}&1\end{bmatrix}$, then find the matrix $A^{50}$

57) If $A=\begin{bmatrix}1&0\\-1&1\end{bmatrix}$ then prove that $A^2-2A+I_2=0$. Hence find the matrix $A^{50}$

58) If $A=\begin{pmatrix}1\\2\\3\end{pmatrix}$ then find $AA^T$

59) If $A=\begin{pmatrix}1&2&3\end{pmatrix}$ then find $AA^T$

60) If $P=\begin{pmatrix}1&2&1\\1&3&1\end{pmatrix}$ and $Q=PP^T$, then find the matrix $Q$

61) If $A=\begin{pmatrix}-2&1&3\\0&4&-1\end{pmatrix}$ and $B=\begin{pmatrix}2&1\\-3&0\\4&-5\end{pmatrix}$, then prove that $(AB)^T=B^TA^T$

62) If $A=\begin{bmatrix}1&2&3&4\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ then find $AB$ and $BA$

63) Express the following matrix ($A$) as the sum of symmetric & skew-symmetric matrix.

$$A=\begin{bmatrix}-3&4&1\\2&3&0\\1&4&5\end{bmatrix}$$

64) Express the following matrix ($A$) as the sum of symmetric & skew-symmetric matrix.

$$A=\begin{bmatrix}2&3&4\\5&7&9\\-2&1&1\end{bmatrix}$$

65) If $A=\begin{bmatrix}1&x\\x^2&4y\end{bmatrix}$, $B=\begin{bmatrix}-3&1\\1&0\end{bmatrix}$ and $adj(A)+B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ then find the value of $x$ and $y$.

66) If $A=\begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}$ then show that $A^2-5A-2I_2=O$ where $I_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $O=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$. Hence find $A^{-1}$

67) If $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$ and $a$, $b$ are two real numbers such that $A^2+aA+bI=O$ (where $I$ & $O$ are unit & zero matrix), then find $A^{-1}$

68) If $A=\begin{pmatrix}2x & 0\\ x & x\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}1 & 0\\ -1 & 2\end{pmatrix}$, then find the value of $x$

69) If $A=\begin{pmatrix}2 & -3\\ -4 & 7\end{pmatrix}$ and $2A^{-1}=kI-A$ where $I$ is unit matrix. Find the value of $k$.

70) If $A=\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix}$ then show that $6A^{-1}+5I=A$

71) If $A^{-1}=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$ then find the matrix $A$

72) If $A=\begin{bmatrix}4 & 1\\ 7 & 2\end{bmatrix}$ then find the matrix $B$ such that $AB=I$ where $I$ is the unit matrix of order 2.

73) If $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$ and $AB=\begin{bmatrix}-13 & 8\\ -8 & 5\end{bmatrix}$, then find the matrix $B$.

74) Find a matrix $A$ such that $A\begin{bmatrix}4 & -2\\ 0 & 5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ -9 & 6\end{bmatrix}=\begin{bmatrix}3 & 16\\ 7 & 8\end{bmatrix}$

75) If $A=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 1\end{bmatrix}$ then find the value of $(AB)^{-1}$

76) If $A=\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & -2\\ -1 & 1\end{pmatrix}$ then show that $(AB)^{-1}=B^{-1}\cdot A^{-1}$

77) If $A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 2\end{bmatrix}$, then find $A^{-1}\cdot B$

78) If inverse of $A=\begin{pmatrix}k & 2\\ 3 & 4\end{pmatrix}$ does not exist, then find the value of $k$.

79) If $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$ then show that $A(adjA)=|A|\cdot I$

80) If $A=\begin{bmatrix}2 & -1\\ -4 & 3\end{bmatrix}$, then justify the following relation: $\left(A^2\right)^{-1}=\left(A^{-1}\right)^2$

81) If $A=\begin{pmatrix}1 & 1\\ 2 & 2\\ 3 & 3\end{pmatrix}$ then find $AA^{-1}$

82) If $A=\begin{bmatrix}k & 2\\ 2 & k\end{bmatrix}$ and $\left|A^3\right|=125$, then find value of $k$.

83) Find the matrices $A$ and $B$ such that, $A+2B=\begin{pmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{pmatrix}$ and

$2A-B=\begin{pmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{pmatrix}$

84) Given $A=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ and $B=\begin{bmatrix}2 & x & x\\ x & 4 & 5\\ x & 6 & 7\end{bmatrix}$. Then find the value of $x$ (if possible) such that $AB=BA$

85) If $A=\begin{pmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{pmatrix}$ and $A^TA=I$, then find the value of $a, b, c$

86) If $A=\begin{bmatrix}1 & -2 & 2\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 0\\ 2 & 3 & -1\\ 0 & -1 & -2\end{bmatrix}$, then show that the matrix $\left(A^TB\right)A$ is a diagonal matrix.

87) Show that the matrix $A=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is an idempotent matrix.

88) Show that the matrix $\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$ is a Nilpotent matrix of index 3.

89) If $A=\begin{bmatrix}1 & x & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{bmatrix}$ and $A^2+2I_3=3A$, then find the value of $x$. Here $I_3$ is the unit matrix of order 3.

90) If $P = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & -5 \end{bmatrix}$ then show that $P^2 = P$ and then find a matrix $Q$ such that $3P^2 - 2P + Q = I$.

91) Show that matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I_3 = O$. Hence find $A^{-1}$

92) If $f(x) = \begin{bmatrix} cos\ x & -sin\ x & 0 \\ sin\ x & cos\ x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(\alpha) \cdot f(\beta) = f(\alpha + \beta)$.

93) If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I$ is the identity matrix of order 2, then show that $(I + A) = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ (or) $(I + A)(I - A)^{-1} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

94) If $\alpha - \beta = (2n + 1)\frac{\pi}{2}$ where $n \in \mathbb{Z}$ then show that product of $\begin{pmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix}$ and $\begin{pmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{pmatrix}$ is a zero matrix.

95) If $A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ then prove that $AB^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

96) If $A = \begin{pmatrix} 1 & \tan\ x \\ -\tan\ x & 1 \end{pmatrix}$ then show that $A^T \cdot A^{-1} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$

97) If a matrix $A = \begin{pmatrix} 6 & 2 & -2 \\ -2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$, then show that $(A - 2I)(A - 4I) = 0$ matrix and hence find $A^3$

98) Find the value of $\begin{pmatrix} x & y & z \end{pmatrix} \times \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

99) Find the value of $p$ such that $\begin{bmatrix} 1 & p & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ p \end{bmatrix} = 0$

100) If $f(x) = x^2 - 5x + 6$ then find $f(A)$ where $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

101) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then show that $A$ is a root of $f(x) = x^3 - 6x^2 + 7x + 2$

102) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is proper orthogonal, and hence find $A^{-1}$

103) Show that the matrix $\begin{pmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \\ 3 & 6 & 6 \end{pmatrix}$ is non-singular.

104) If $|A| = 2$ and $Adj(A) = \begin{bmatrix} -2 & 3 & 1 \\ 6 & -8 & -2 \\ -4 & 7 & 2 \end{bmatrix}$ then find the matrix $A$

105) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & -1 \\ 0 & 5 & 6 \end{bmatrix}$ then show that $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

106) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, then show that $A^{-1} = A^T$.

107) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then show that $A^2 = A^{-1}$. Hence find $A^3$

108) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify the relation $A \cdot (adj\ A) = |A| \cdot I$. Hence find $A^{-1}$

109) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ then find $A^{-1}$ and also show that $AA^{-1} = A^{-1}A = I$

110) If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ then find $A^{-1}$. Hence solve the following equations.
$2x-3y+5z=11$, $3x+2y-4z=-5$, $x+y-2z=-3$

111) If $A=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & -1\\ -1 & 1 & -7\end{bmatrix}$ find $A^{-1}$. Hence solve the following equations. $x+y-z=3$, $2x+3y+z=10$, $3x-y-7z=1$

112) From the matrix equation $AX=B$ find the matrix $X$, given that $A=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 1 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2\\ 1\\ 7\end{bmatrix}$

113) Find the matrix $A$ where $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$

114) Solve by using inverse matrix method. $x+2y+z=7$, $x+3z=11$, $2x-y=1$

115) Solve by using inverse matrix method. $2x+3y+5z=5$, $x-2y+z=-4$, $3x-y-2z=3$

116) Solve by using inverse matrix method. $\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$, $\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$

117) By using the matrix method, show that the following set of equations have an infinite number of solutions. $x+2y+3z=1$, $3x+4y+5z=2$, $5x+6y+7z=3$

118) Given $A=\begin{bmatrix}1 & -1 & 1\\ 0 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -3\end{bmatrix}$. Find $AB$. Hence from this result, solve the following set of equations. $x-y+z=4$, $x-2y-2z=9$, $2x+y+3z=1$

119) Find the inverse of the following matrix by using elementary row transformations.
$\begin{pmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{pmatrix}$

120) Find the inverse of the following matrix by using Elementary row operations.
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

121) Find the inverse of the following matrix by using Elementary column operations.
$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

122) For any square matrix $A$, wrong statement is -- (a) $(adj\ A)^{-1} = adj\ \left(A^{-1}\right)$, (b) $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$, (c) $\left(A^3\right)^{-1} = \left(A^{-1}\right)^3$, (d) none of these.

123) $A$ and $B$ are two matrices such that $AB = BA$. Then show that $A^2 + B^2 = A + B$

124) Prove that any square matrix can be expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

125) If $A$ is a square matrix, then show that the matrices $A \cdot A^T$ and $A^T \cdot A$ are symmetric.

126) If $A$ and $B$ are two symmetric matrices of same order, then show that $(AB - BA)$ is a skew-symmetric matrix.

127) If $A$ is a skew symmetric matrix, then show that $A^2$ is a symmetric matrix.

128) If $A$ is a skew-symmetric matrix and $I + A$ isa non-singular matrix, then show that $(I - A)(I + A)^{-1}$ is an orthogonal matrix.

129) If $A$ and $B$ are two square matrices of same order and $A^{-1}$, $B^{-1}$ exist. Then prove that inverse of $AB$ also exists and $(AB)^{-1} = B^{-1}A^{-1}$.

130) Prove that inverse of any square matrix (if exists) is unique.