# Complex Number

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1) Covert the following complex number into $A + iB$ form --

i) $\left(\frac{1+i}{1-i}\right)^3$

ii) $\left(\frac{1-i}{1+i}\right)^{100}$

iii) $\frac{(1+i)^2}{3-i}$

iv) $\frac{\sqrt{3}+i\sqrt{2}}{2\sqrt{3}+i\sqrt{2}}$

v) $\frac{1}{1-\cos\theta - i\sin\theta}$

2) If $\frac{2+i}{2-3i} = A + iB$ then find the value of $A^2 + B^2$

3) If $x + iy = \frac{2}{3+\cos\theta + i\sin\theta}$ then show that $2x^2 + 2y^2 = 3x - 1$

4) If $(x+iy)^5 = a + ib$ then find the value of $(y+ix)^5$

5) Find conjugate of the following complex numbers --

i) $-\sqrt{5} + 7i$

ii) $\frac{1}{1+i}$

iii) $\frac{2-i}{(1-3i)^2}$

6) Which one is correct? i) $2+3i > 1+4i$, ii) $3+3i > 6+2i$, iii) $5+9i > 5+6i$, iv) none of these.

7) For any complex number $z$, show that $|z| > \frac{|\text{Re}(z)| + |\text{Im}(z)|}{\sqrt{2}}$

8) If $z_1$ and $z_2$ are two complex numbers, then show that--

i) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$

ii) $2(z_1+z_2) \geq (|z_1|+|z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right|$

9) Find the least positive whole number of $n$, that satisfies the equation $\left(\frac{1+i}{1-i}\right)^n = 1$

10) If $(1+i)(2+i)(3+i)\cdots(n+i) = a+ib$, then show that $2\cdot 5\cdot 10\cdots\left(n^2+1\right) = a^2+b^2$

11) Find the condition to become purely real & purely imaginary of the expression $(a+ib)\cdot(c+id)$

12) Find the modulus of following complex numbers --
i) $\frac{1}{1-i}$
ii) $\frac{-i}{1+i}$
iii) $\frac{1-i}{\sqrt{5}-i\sqrt{3}}$
iv) $(a-ib)^2$
v) $(3i-1)^2$

13) Find argument or principal value of amplitude of following complex numbers --
i) $3i$
ii) $-3-\sqrt{3}i$
iii) $\frac{1}{1+i}$
iv) $-2$

14) Find modulus & amplitude of following complex numbers --
i) $1+i\tan\frac{3\pi}{5}$
ii) $1+i\tan\frac{4\pi}{7}$

15) Express the following complex numbers into modulus-amplitude form --
i) $\sqrt{3}-i$
ii) $\frac{i}{1-i}$
iii) $\frac{\sqrt{3}-i}{1-\sqrt{3}i}$

16) If $iz^2-\bar{z}=0$, then find $|z|$

17) If $8iz^3+12z^2-18z+27i=0$ then find $|z|$

18) If $\left|z-\frac{6}{z}\right|=2$ then find $|z|$, where $|z|$ is a complex number.

19) If $z$ is a complex number, then find the minimum value of $|z|+|z-1|$

20) If $z$ is a complex number and $\left|z+\dfrac{2}{z}\right|=2$, then find maximum value of $|z|$

21) If $z$ is a complex number and $|z-2|\leq 3$, then find maximum value of $|z|$

22) If $z$ is a complex number and $|z+4|\leq 3$, then find the maximum value of $|z+1|$

23) If $z$ is a complex number and $|z+2|\leq 2$, then find maximum & minimum value of $|z|$

24) If $z$ is a complex number and $|z+5|\leq 6$, then find maximum & minimum value of $|z+2|$

25) If $z$ is a complex number and $|z+2|+|z-2|\leq 6$, then find maximum value of $|z|$

26) $z$ is a complex number and $\left|z^2-9\right|=8|z|$, then find maximum & minimum value of $|z|$

27) $z, z_0$ are complex numbers and $|z-i|\leq 2$, $z_0=5+3i$; then find the maximum value of $|iz+z_0|$

28) If $z_1, z_2, z_3$ are three complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+\dfrac{1}{z_3}\right|=1$, then find the value of $|z_1+z_2+z_3|$

29) $z$ is a complex number such that $|z|=4$ and $arg(z)=\dfrac{5\pi}{6}$. Find the complex number $z$

30) If $z=1+i\tan\alpha$ (where $\pi<\alpha<\dfrac{3\pi}{2}$), then find $|z|$.

31) If $z_1=3i$ and $z_2=-1-i$, then find $arg\left(\dfrac{z_1}{z_2}\right)$

32) If $z_1=\dfrac{1-i}{\sqrt{2}}$, $z_2=\dfrac{1+i\sqrt{3}}{2}$, then find the principal value of argument of $z_1z_2$

33) If $z_1, z_2$ are two complex numbers and $|z_1+z_2|=|z_1|+|z_2|$, then prove that $\arg(z_1)=\arg(z_2)$

34) If $z_1, z_2$ are two complex numbers and $|z_1+z_2|=|z_1-z_2|$, then find the value of $\operatorname{amp}(z_1)-\operatorname{amp}(z_2)$

35) If $z_1, z_2$ are two complex numbers and $|z_1|=|z_2|=1$ and $\arg(z_1)+\arg(z_2)=0$. Then show that $z_1=\dfrac{1}{z_2}$

36) If $z_1, z_2$ are two complex numbers and $|z_1| = |z_2|$ and $\arg(z_1)$~$\arg(z_2) = \pi$, then show that $z_1 + z_2 = 0$

37) Prove that, $\operatorname{amp}(z) - \operatorname{amp}(-z) = \pm\pi$ when $\operatorname{amp}(z)$ be positive or negetive.

38) Find the value of --

i) $1 + i + i^2 + i^3 + i^4$

ii) $1 + i^2 + i^4 + i^6 + \cdots + i^{16}$

iii) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ where $n \in \mathbb{N}$

iv) $\dfrac{i + i^2 + i^3 + i^4}{1 + i}$

v) $\dfrac{i^2 + i^4 + i^6 + i^7}{1 + i^2 + i^3}$

vi) $\displaystyle\sum_{n=0}^{225} i^n$

vii) $(1 + i)^6\left(1 + \dfrac{1}{i}\right)^6$

viii) $\sqrt{-3 + \sqrt{-3 + \sqrt{-3 + \cdots \infty}}}$

39) i) If $a = \dfrac{1 + i}{\sqrt{2}}$ then show that $1 + a^2 + a^4 + a^6 = 0$

ii) If $x = 2 - i\sqrt{3}$ then find the value of $2x^4 - 5x^3 - 3x^2 + 41x - 35$

iii) If $x = 2 + 3i$ then find the value of $x^3 - 4x^2 + 13x + 5$

iv) If $x = -1 + i\sqrt{2}$ then find the value of $x^4 + 4x^3 + 6x^2 + 4x + 9$

40) Find square root of following complex number --

i) $i$

ii) $-2i$

iii) $\dfrac{1 + i}{1 - i}$

iv) $\dfrac{-1 + \sqrt{3}}{2}$

v) $\dfrac{-1 + \sqrt{-3}}{2}$

vi) $7 - 24i$

vii) $y + \sqrt{y^2 - x^2}$ where $y^2 < x^2$

viii) $1 + i\sqrt{a^4 - 1}$

viii) $x - i\sqrt{x^4 + x^2 + 1}$

41) Show that one of the values of $\sqrt{i} + \sqrt{-i}$ is $\sqrt{2}$

42) Show that one of the values of $\sqrt{1 + i} - \sqrt{1 - i}$ is $i\sqrt{2\left(\sqrt{2} - 1\right)}$

43) If $y=\sqrt{x^2+6x+8}$ where $x>0$, then show that one of the values of $\sqrt{1+iy}+\sqrt{1-iy}$ is $\sqrt{2x+8}$

44) i) If $z=x+iy$ where $x,y\in\mathbb{R}$ and $i=\sqrt{-1}$ and $|z-2|=|2z-1|$, then prove that $x^2+y^2=1$
ii) If $z=x+iy$ where $x,y\in\mathbb{R}$ and $i=\sqrt{-1}$ and $|z+6|=|2z+3|$, then prove that $x^2+y^2=9$

45) If $z=x+iy\ (x,y\in\mathbb{R})$ and $i=\sqrt{-1}$ and $|2z+1|=|z-2i|$ then show that $3(x^2+y^2)+4(x+y)=3$

46) If $z=3+2i$ and $\dfrac{2z-1}{z-2}=x+iy$ (where $x,y$ are real), then find the value of $x,y$.

47) If $a,b,c,d,x,y$ are real number and $(a+ib)(c+id)=(x+iy)$, then show that $(ac-bd)^2+(ad+bc)^2=x^2+y^2$

48) If $a^2+b^2=1$ (where $a,b\in\mathbb{R}$), then show that a real value of $x$ satisfies the equation $\dfrac{1-ix}{1+ix}=a-ib$

49) If a complex number $z$ is such that $\dfrac{z-1}{z+1}$ is purely imaginary, then show that $|z|=1$

50) If a complex number $z$ is such that that $|z|=1$, then show that $\dfrac{z-1}{z+1}$ is purely imaginary.

51) If $x+iy=\sqrt{\dfrac{a+ib}{c+id}}$, then show that $\left(x^2+y^2\right)^2=\dfrac{a^2+b^2}{c^2+d^2}$

52) If $\sqrt{x-iy}=a-ib$ (where $x,y,a,b\in\mathbb{R}$) then show that $\sqrt{x+iy}=a+ib$

53) i) If $(a-ib)^{\frac{1}{3}}=p-iq$ (where $a,b,p,q\in\mathbb{R}$) then show that $(a+ib)^{\frac{1}{3}}=p+iq$
ii) If $(x-iy)^{\frac{1}{3}}=a-ib$ (where $x,y,a,b\in\mathbb{R}$) then show that $\dfrac{x}{a}+\dfrac{y}{b}=4\left(a^2-b^2\right)$

54) If $x=\cos\theta+i\sin\theta$ (where $\theta$ real), then show that $x^2+\dfrac{1}{x^2}$ is a real number.

55) If $x-\dfrac{1}{x}=2i\sin\theta$, then show that $x^4-\dfrac{1}{x^4}=2i\sin 4\theta$

56) If $a=\cos\alpha+i\sin\alpha$ and $b=\cos\beta+i\sin\beta$ then show that $\dfrac{a}{b}+\dfrac{b}{a}=2\cos(\alpha-\beta)$

57) If $x = \cos\theta + i\sin\theta$ (where $\theta$ real) and $1+\sqrt{1-a^2} = na$, then show that $\frac{a}{2n}(1+nx)\left(1+\frac{n}{x}\right) = 1 + a\cos\theta$

58) The three vertices of an equilateral triangle are represented by three complex numbers $z_1, z_2, z_3$. Then show that --
i) $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$
ii) $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

59) The three points in a complex plane are represented by $z_1, z_2, z_3$ such that $|z_1| = |z_2| = |z_3|$ and they form an equilateral triangle in complex plane. Prove that $z_1 + z_2 + z_3 = 0$

60) In complex plane, three points are represented by three complex numbers $z_1, z_2, z_3$ and $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$. Then find the area of triangle formed by these three points.

61) In a complex plane three points, represented by three complex numbers $z_1, z_2, z_3$ and $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$. Show that the triangle formed by these three points are equilateral triangle.

62) In a complex plane, the three points $z_1, z_2, z_3$ are three vertices of an isosceles right angle triangle with right angle at $z_3$. Then show that $z_1^2 + 2z_2^2 + z_3^2 = 2z_2(z_1 + z_3)$

63) $z_1, z_2$ are two complex numbers and $z_1^2 + z_2^2 + 2z_1 z_2 \cos\theta = 0$. Then show that the triangle formed by origin, $z_1$ and $z_2$ is an isosceles triangle. (where $\theta \in \mathbb{R}$)

64) If $z = x + iy$, $w = \frac{1 - iz}{z - i}$ and $|w| = 1$, then show that $z$ lies on real axis on the complex plane.

65) Show that the three points $1 + 4i$, $2 + 7i$ and $3 + 10i$ are collinear.

66) If three points $z, \ -iz$ and 1 are collinear, then show that $z$ always lies on a circle.

67) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that locus of $z$ in complex plane is a circle.

68) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$, then show that locus of $z$ in a complex plane is a circle.

69) If $z_1 = 4 + 3i$, $z_2 = 7 + 4i$ and $z$ is another complex number such that $\arg\left(\frac{z_1 - z}{z - z_2}\right) = \frac{\pi}{4}$; then show that $|z - 6 - 2i| = \sqrt{5}$

70) If $z = x + iy$ (where $x, y \in \mathbb{R}$) and $\left|\frac{z-3}{z+3}\right| = 2$, then find the locus of $z$ in complex plane.

71) If $z = x + iy$ and $\frac{z-i}{z-1} = ia$ (where $x, y, a$ are real), then show that
$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

72) If $z = x + iy$ ($x, y \in \mathbb{R}$) and $\frac{z+1}{z+i}$ is purely imaginary, then show that locus of $z$ is a circle with centre at $-\frac{1}{2}(1+i)$ and radius $\frac{1}{\sqrt{2}}$

73) If $z = x + iy$ and $|z - 2 - i| = 5$, then show that locus of $z$ in complex plane is a circle. Also find its centre & radius.

74) If $z = x + iy$ then find the numerical value of area of circle determined by
$z\bar{z} + (3-4i)z + (3+4i)\bar{z} = 0$

75) If $a, b, c$ real number, $z$ complex number and $a^2 + b^2 + c^2 = 1$, $b + ic = z(1+a)$ then show that $\frac{a+ib}{1+c} = \frac{1+iz}{1-iz}$

76) Find the value of --
i) $(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^5\right)$
ii) $\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)\left(1-\omega^{10}\right)$
iii) $\left(3+\omega+3\omega^2\right)^4$
iv) $1+\omega^{28}+\omega^{29}$
v) $\omega^4+\omega^8+\omega^{-1}\cdot\omega^{-2}$
vi) $\omega^{3n}+\omega^{3n+1}+\omega^{3n+2}$ where $n \in \mathbb{N}$
vii) $\left(\frac{-1+\sqrt{-3}}{2}\right)^{19} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{19}$
viii) $\left(x+y\omega+z\omega^2\right)^2 + \left(x\omega+y\omega^2+z\right)^2 + \left(x\omega^2+y+z\omega\right)^2$
ix) $1\cdot(2-\omega)\cdot\left(2-\omega^2\right) + 2\cdot(3-\omega)\cdot\left(3-\omega^2\right) + \cdots + (n-1)\cdot(n-\omega)\cdot\left(n-\omega^2\right)$

77) Find the value of $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}$ where $\alpha, \beta$ are imaginary cube root of 1.

78) Show that the value of $\left(a+\omega+\omega^2\right)\left(a+\omega^2+\omega^4\right)\left(a+\omega^4+\omega^8\right)\cdots$ upto $2n$ number of factors is $(a-1)^{2n}$

79) Show that $\frac{\omega}{9}\left[(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right) + 9\left(\frac{c+a\omega+b\omega^2}{a\omega^2+b+c\omega}\right)\right] = -1$

80) Find the roots of the equation $(x+5)^3+27=0$

81) Let $\omega$ be the imaginary cube root of 1. If $x=a+b$, $y=a\omega+b\omega^2$, $z=a\omega^2+b\omega$, then show that --
i) $xyz=a^3+b^3$
ii) $x^3+y^3+z^3=3\left(a^3+b^3\right)$
iii) $x^2+y^2+z^2=6ab$

82) Let $\omega$ be the imaginary cube root of 1. If $x=\alpha+\beta$, $y=\alpha+\beta\omega$, $z=\alpha+\beta\omega^2$ , then show that $x^3+y^3+z^3=3\left(\alpha^3+\beta^3\right)$

83) Let $\omega$ be the imaginary cube root of 1 and $a+b+c=0$. Then show that
$\left(a+b\omega+c\omega^2\right)^3+\left(a+b\omega^2+c\omega\right)^3=27abc$